Mixing by internal gravity waves in stars: assessing numerical simulations against theory

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Context

We require **extra mixing atop convective cores**, as suggested by
- ❑ colour-magnitude/Hertzsprung-Russell diagrams, [1]
- ❑ surface chemical abundances, [2]
- ❑ asteroseismology [3]

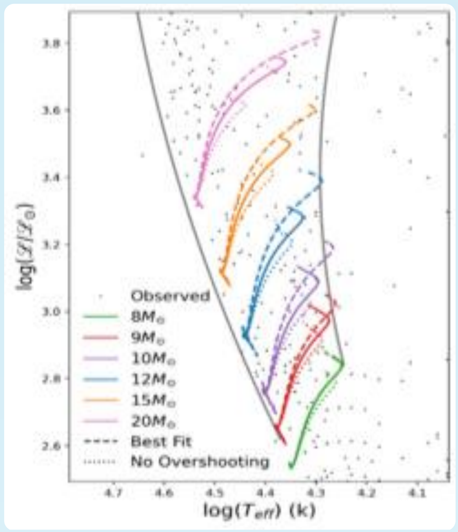
Convective **overshooting is not strong enough by itself**, [4] especially in stars evolved beyond the zero-age main-sequence (ZAMS), where the Helium stratification hinders radial motion. [5]

From numerical simulations, we find **overshooting is reduced by over 60%** by the end of the main-sequence.

Model	l_{ov}/H_p
ZAMS	8.3%
Mid MS	2.8%
TAMS	2.8%

Evolution

l_{ov} — Overshooting length
H_p — Pressure scale height



Baraffe et al. 2023 — evolutionary models with/without convective overshooting, vs. observations.

Since overshooting is not enough, we turn to **internal gravity waves (IGWs)** as a potential source of additional mixing.

IGW Mixing Mechanisms

❑ Via non-restorative effects of **thermal diffusion.** [7]

The fluid loses a small portion of its entropy to the surroundings during each wave period, displacing its equilibrium.

$$D_{P81}(\omega, r, \ell) = \frac{\epsilon^4 \kappa_T^2 k_v^2}{\omega}$$

with *nonlinearity parameter* $\epsilon = \frac{k_v u_v}{\omega} \lesssim 1$.

❑ Via **sub-wavelength horizontal shearing** motions, leading to Kelvin-Helmholtz instabilities. [8]

This process is enhanced by thermal diffusion, which lowers the threshold for instability.

$$D_{GLS91}(\omega, r, \ell) = \frac{[\ell(\ell+1)]^{3/2}}{4\rho r^3} \frac{N}{\omega^4} \kappa_T F_{wave}$$

with *wave flux* $F_{wave} = \rho v^2 \cdot u_{g,r}$.

❑ **Wave breaking** will create efficient mixing, but IGWs are **typically linear** atop convective cores. [7]

There is *little evidence that any of these mechanisms work in stars*. We will **compute these theoretical predictions for our models and compare with numerical results**.

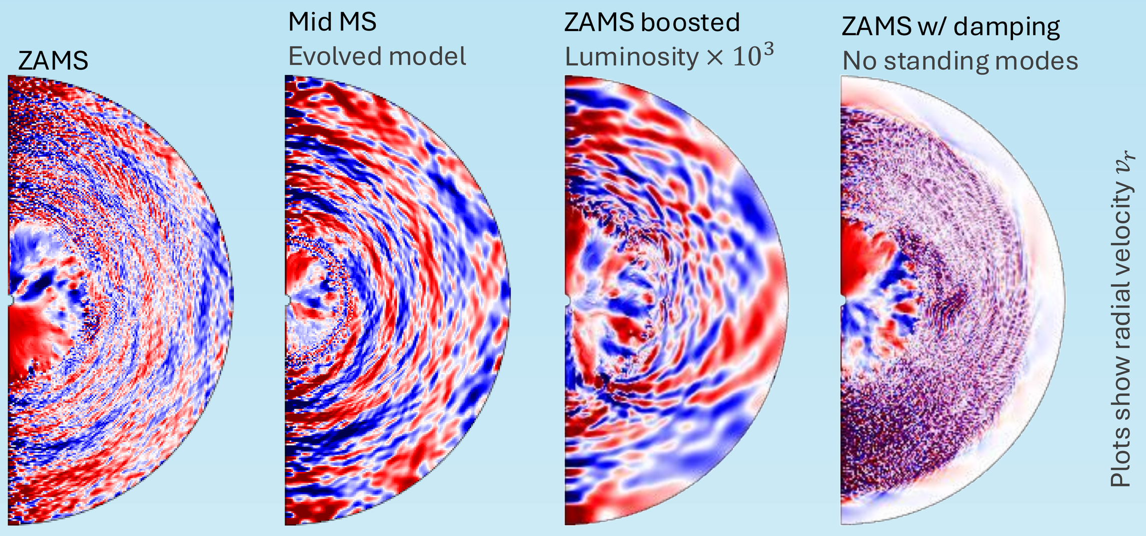
Numerical Simulations

We perform 2D stellar simulations with the **Multidimensional Stellar Implicit Code (MUSIC)** [6] of a $20M_\odot$ star at the ZAMS and mid-MS evolutionary stages, up to 80% of the star's radius.

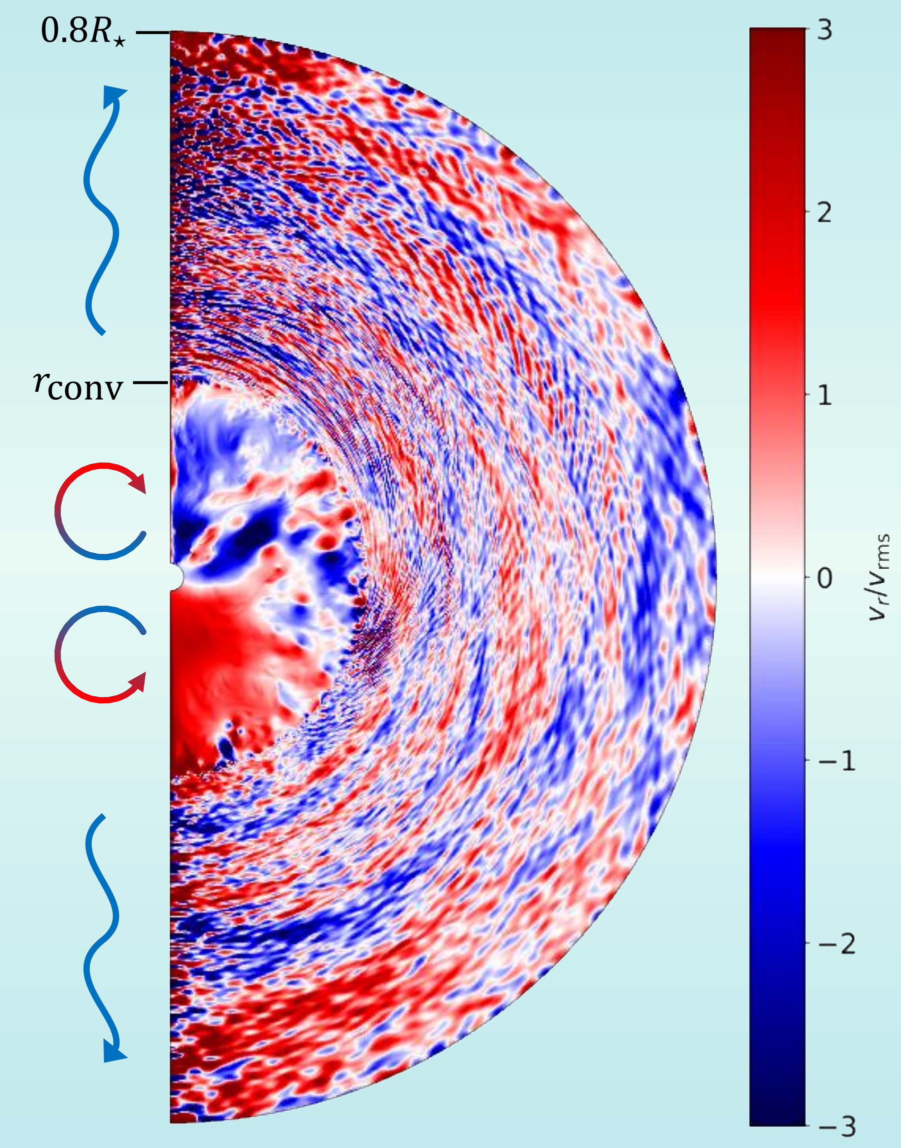
We compute **power spectra** $P[\hat{v}_r]$ of the vertical velocity from simulations, to obtain the power of **each mode** (ω, ℓ). From this we compute the diffusion coefficients D_{P81} and D_{GLS91}.

$$v_r(t, r, \theta) \xrightarrow{\text{Fourier Transform} \,/\, \text{Spherical Harmonics}} P[\hat{v}_r](\omega, r, \ell)$$

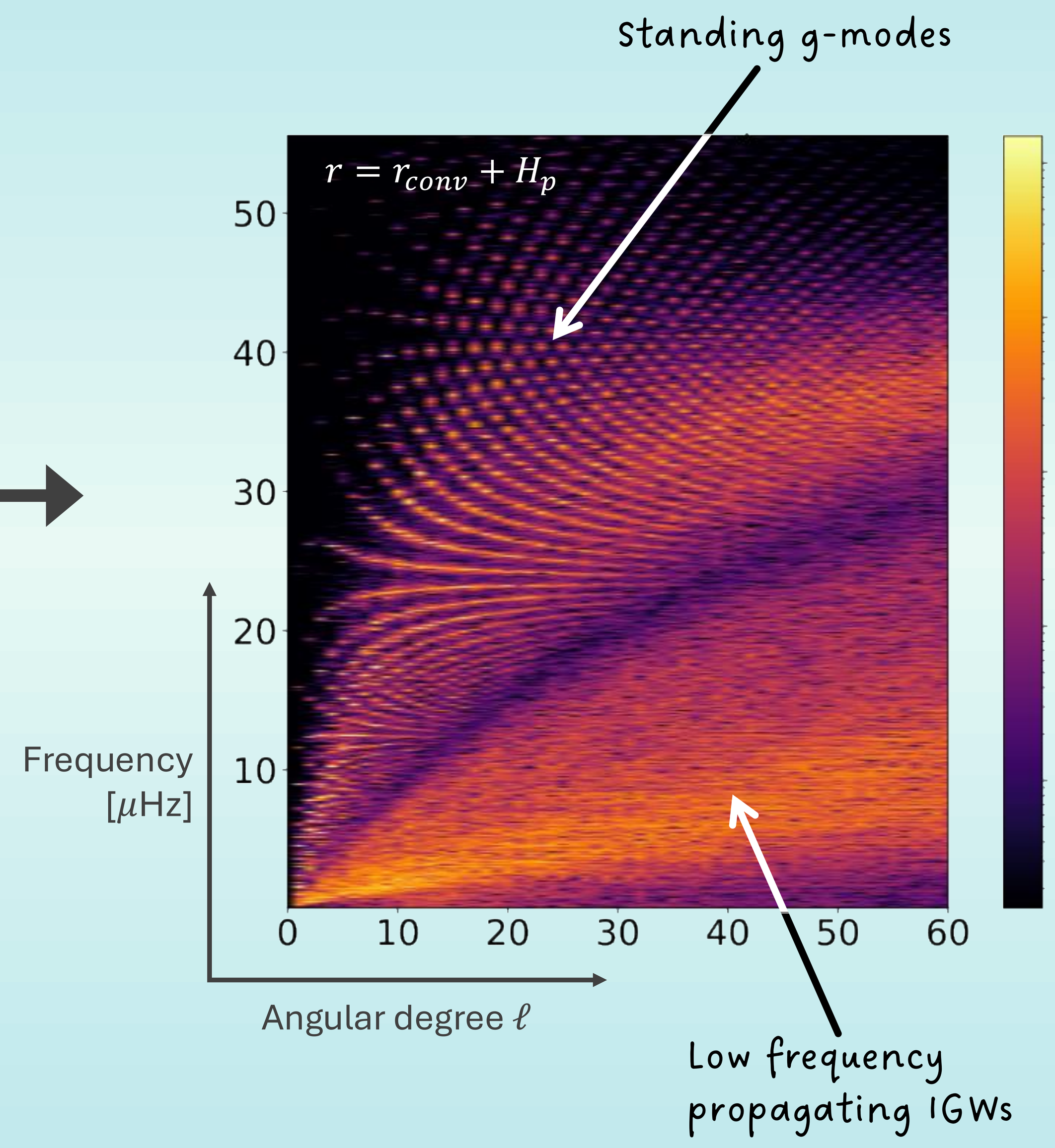
We also run a simulation with a **boosted luminosity** and **thermal diffusivity**, scaled by 10^3. Larger velocities and higher frequencies give altered wave spectra.
A simulation with a **damping layer** at the outer boundary allows us to study the propagating waves independently of g-modes.



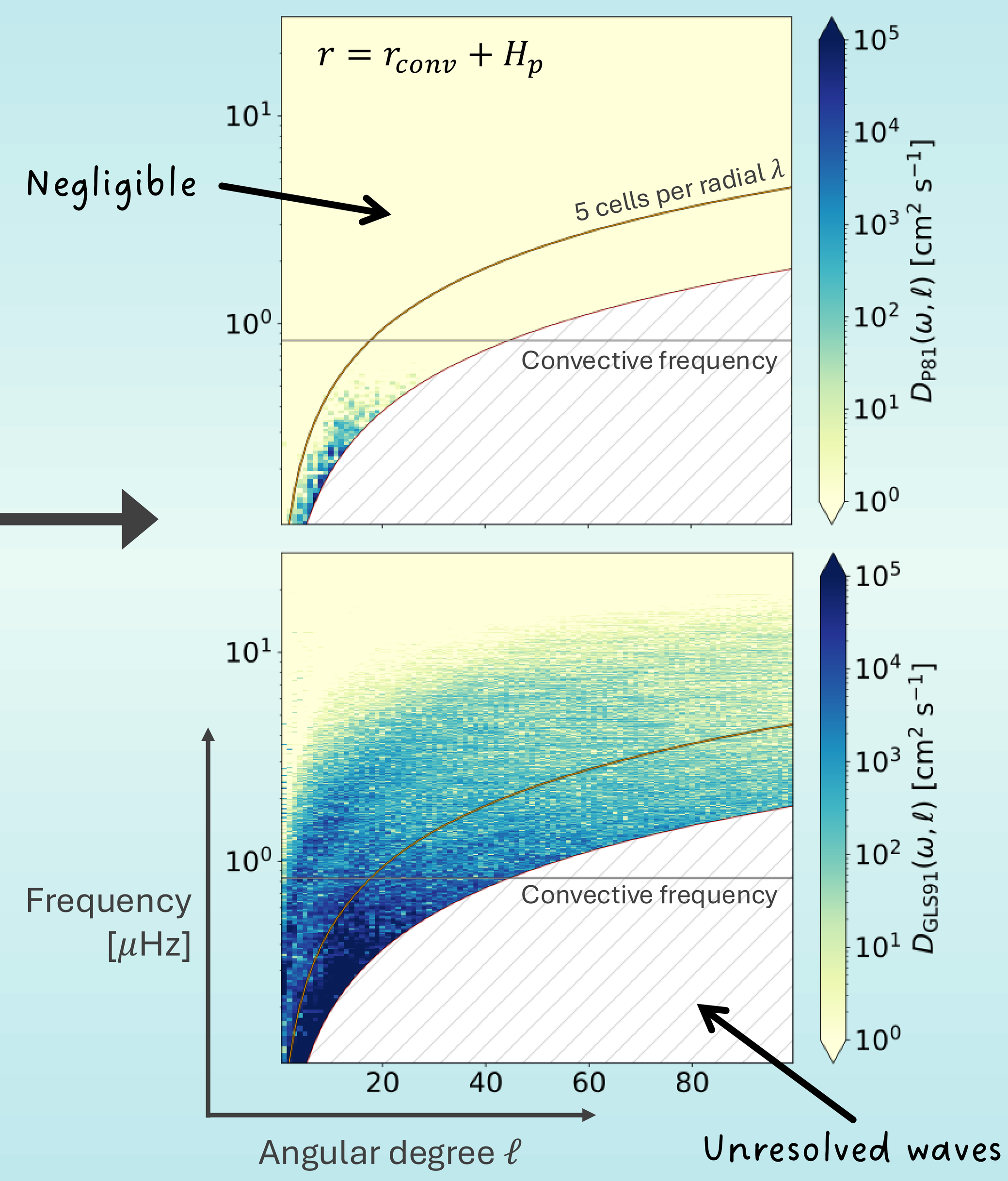
1. Numerical simulations



2. Power spectra

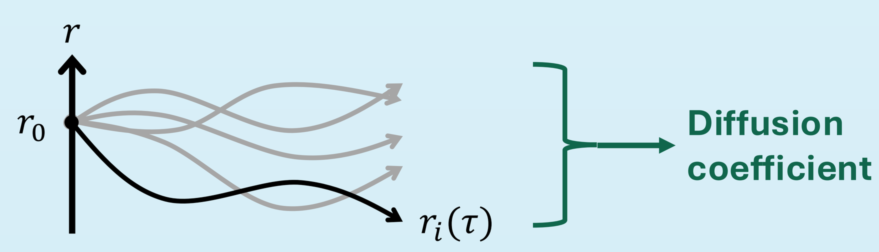


3. Diffusion coefficients



Measuring With Tracer Particles

Lagrangian particles are advected by the velocity field at each timestep in the simulation. We compute a diffusion coefficient from the **mean squared radial displacement** of particles over time. [9]



For the simulation with artificially boosted luminosity, the tracer particles yield **diffusion coefficients smaller than previous studies** by up to two orders of magnitude. [9]
Additionally, the process does **not appear diffusive**.

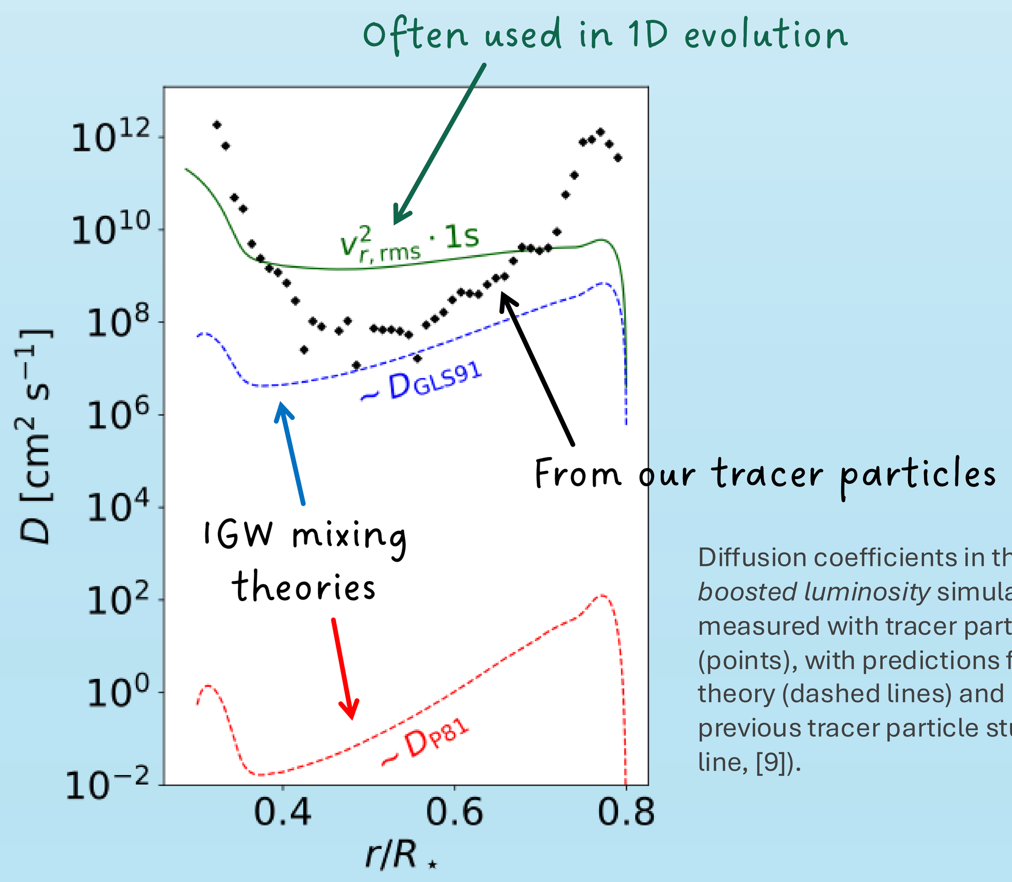
For simulations with "realistic" diffusivities and viscosities, we find tracers particle methods are **prone to numerical artefacts**, since they rely on:

- ❑ **Small timesteps**
  to resolve *high frequency* IGWs
- ❑ **Long simulations**
  to capture *diffusive timescales*
- ❑ A **high resolution**
  to resolve IGWs with *short radial wavelengths*
- ❑ **Sophisticated interpolation** of velocities
  to capture Lagrangian displacements between grid points.

In Conclusion...

- ❑ Both **theories predict small diffusion coefficients** immediately above the convective core of our non-boosted simulations.
  - ❑ Irreversible effects due to thermal diffusion (D_{P81}) are **negligible**.
  - ❑ Sub-wavelength shearing gives $D_{GLS91} \sim 10^3 \mathrm{cm^2 s^{-1}}$, and **smaller for more evolved models**, below what can be reliably measured by tracer particles.
- ❑ Both theories consider **monochromatic waves**, but the IGWs in stars have a broad spectrum. Attempts to include this in theories (e.g. [10]) are limited.
- ❑ **Tracer particles** in stellar simulations are often **prone to numerical issues**, due to the range of spatial and temporal scales.
- ❑ Diffusion coefficients from tracer particle methods are **used without scrutiny** in 1D stellar evolution codes e.g. [11]
- ❑ Where tracer particles do work in a simulation with *artificially enhanced luminosity*, we find **slower mixing than previous studies**. Resulting diffusion coefficients cannot be scaled back to non-enhanced cases. [12]

Commonly quoted theories for chemical mixing by IGWs in stars act **very slowly**.
Numerical results from tracer particle methods **do not match** with these theories and are **prone to numerical artefacts** for "realistic" wave spectra.



Diffusion coefficients in the *boosted luminosity* simulation, measured with tracer particles (points), with predictions from theory (dashed lines) and a previous tracer particle study (solid line, [9]).

What's Next?

Connecting theory with numerical simulations in setups with reduced complexity:
- ❑ Monochromatic/enforced IGW spectra
- ❑ Small domains with high resolutions

Can we see these theoretical mechanisms working?

How do they apply to a spectrum of waves, and g-modes?

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Read the paper:
Morton et al. 2025
10.1093/mnras/staf010

References
[1] Rosenfield et al. 2017
[2] Brott et al. 2011
[3] Burssens et al. 2023
[4] Baraffe et al. 2023
[5] Morison et al. 2024
[6] Viallet, Baraffe & Walder 2011, Viallet et al. 2016, Goffrey et al. 2017
[7] Press 1981
[8] García Lopez & Spruit 1991
[9] Rogers & McElwaine 2017
[10] Montalbán 1994, Montalbán & Shatzman 1996, 2000
[11] Pedersen et al. 2018, Li et al. 2023, Michielsen et al. 2023
[12] Le Saux et al. 2022

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